

Further Statistics 1 Mark Scheme

Question	Scheme	Marks	AOs																	
1(a)	H ₀ : There is no association between language and gender	B1	1.2																	
		(1)																		
(b)	$\frac{54 \times 85}{150} = 30.6 \quad *$	B1*cs0	1.1b																	
		(1)																		
(c)	<table><tr><th colspan="2" rowspan="2">Expected frequencies</th><th colspan="3">Language</th></tr><tr><th>French</th><th>Spanish</th><th>Mandarin</th></tr><tr><td rowspan="2">Gender</td><td>Male</td><td>26.43...</td><td>23.4</td><td>15.16...</td></tr><tr><td>Female</td><td>34.56...</td><td>[30.6]</td><td>19.83...</td></tr></table>	Expected frequencies		Language			French	Spanish	Mandarin	Gender	Male	26.43...	23.4	15.16...	Female	34.56...	[30.6]	19.83...	M1	2.1
	Expected frequencies			Language																
			French	Spanish	Mandarin															
	Gender	Male	26.43...	23.4	15.16...															
		Female	34.56...	[30.6]	19.83...															
$\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(23 - 26.43)^2}{26.43} + \dots + \frac{(15 - 19.83)^2}{19.83}$ Awrt <u>3.6/3.7</u>	M1	1.1b																		
	A1	1.1b																		
	(3)																			
(d)	Degrees of freedom (3 – 1)(2 – 1) → Critical value $\chi^2_{2,0.01} = 9.210$	M1	3.1b																	
	As $\sum \frac{(O - E)^2}{E} < 9.210$, the null hypothesis is not rejected	A1	2.2b																	
		(2)																		
(e)	Still not rejected since $\sum \frac{(O - E)^2}{E} < \chi^2_{2,0.1} = 4.605$	B1	2.4																	
		(1)																		
(8 marks)																				
Notes:																				
(a)																				
B1: For correct hypothesis in context																				
(b)																				
B1*: For a correct calculation leading to the given answer and no errors seen																				
(c)																				
M1: For attempt at $\frac{(\text{Row Total})(\text{Column Total})}{(\text{Grand Total})}$ to find expected frequencies																				
M1: For applying $\sum \frac{(O - E)^2}{E}$																				
A1: awrt 3.6 or 3.7																				
(d)																				
M1: For using degrees of freedom to set up a χ^2 model critical value																				
A1: For correct comparison and conclusion																				
(e)																				
A1ft: For correct conclusion with supporting reason																				

Question	Scheme	Marks	AOs
2(a)	$-4 = 2 - 5E(X)$	M1	3.1a
	$E(X) = 1.2$		
	$-1 \times c + 0 \times a + 1 \times a + 2 \times b + 3 \times c = 1.2$	M1	1.1b
	$a + 2b + 2c = 1.2$ 1		
	$P(Y \geq -3) = 0.45$ gives $P(2 - 5X \geq -3) = 0.45$ i.e. $P(X \leq 1) = 0.45$	M1	2.1
	$2a + c = 0.45$ 2		
	$2a + b + 2c = 1$ 3	M1	1.1b
	$\begin{pmatrix} 1 & 2 & 2 \\ 2 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 2 & -3 \\ -2 & -3 & 4 \end{pmatrix} \begin{pmatrix} 1.2 \\ 0.45 \\ 1 \end{pmatrix}$ <u>or</u>	M1	1.1b
	e.g. 3 - 2 $\Rightarrow b + c = 0.55$ sub. $2(b + c)$ into 1 $\Rightarrow a = 0.1$ etc		
	$a = 0.1 \quad b = 0.3 \quad c = 0.25$	A1 A1	1.1b 1.1b
		(7)	
(b)	$\text{Var}(Y) = 75 - (-4)^2$ <u>or</u> 59	M1	1.1a
	$[\text{Var}(Y) = 5^2 \text{Var}(X) \text{ implies}] \text{Var}(X) = 2.36$	A1	1.2
		(2)	
(c)	$P(Y > X) = P(2 - 5X > X) \rightarrow P(X < \frac{1}{3})$	M1	3.1a
	$P(X < \frac{1}{3}) = a + c = 0.35$	A1ft	1.1b
		(2)	
(11 marks)			
Notes:			
(a) M1: For using given information to find an expression for $E(X)$ i.e. use of $E(Y) = 2 - 5E(X)$ M1: For use of $\sum xP(X = x) = '1.2'$ M1: For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in a and c M1: For use of $\sum P(X = x) = 1$ M1: For solving their 3 linear equations (matrix or elimination) A1: For any 2 of a, b or c correct A1: For all 3 correct values			

Question 2 notes continued:	
Another method for part (a) is:	
M1:	For using given information to find the probability distribution for Y leading to an expression for $E(Y)$
M1:	For use of $\sum yP(Y = y) = -4$
M1:	For use of $P(Y \geq -3) = 0.45$ to set up the argument for solving by forming an equation in a and c
M1:	For use of $\sum P(Y = y) = 1$
M1:	For solving their 3 linear equations (matrix or elimination)
A1:	For any 2 of a , b or c correct
A1:	For all 3 correct values
(b)	
M1:	For use of $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$ (may be implied by a correct answer)
A1:	For use of $\text{Var}(aX) = a^2 \text{Var}(X)$ to reach 2.36 or exact equivalent
(c)	
M1:	For rearranging to the form $P(X < k)$
A1ft:	0.1' + '025' (provided their a and c and their $a + c$ are all probabilities)
Another method for part (c) is:	
M1:	For comparing distribution of X with distribution of Y to identify $X = -1$ and $X = 0$
A1ft:	'0.1' + '025' (provided their a and c and their $a + c$ are all probabilities)

Question	Scheme	Marks	AOs
3(a)	$X \sim \text{Po}(2.6) \quad Y \sim \text{Po}(1.2)$		
	P(each hire 2 in 1 hour) $= P(X=2) \times P(Y=2) = 0.25104... \times 0.21685...$	M1	3.3
	$= 0.05444... \quad \text{awrt } \underline{0.0544}$	A1	1.1b
		(2)	
(b)	$W = X + Y \rightarrow W \sim \text{Po}(3.8)$	M1	3.4
	$P(W = 3) = 0.20458.... \quad \text{awrt } \underline{0.205}$	A1	1.1b
		(2)	
(c)	$T \sim \text{Po}((2.6+1.2) \times 2)$	M1	3.3
	$P(T < 9) = 0.64819... \quad \text{awrt } \underline{0.648}$	A1	1.1b
		(2)	
(d)	(i) Mean = $np = \underline{2.4}$	B1	1.1b
	(ii) Variance = $np(1 - p) = 2.3904 \quad \text{awrt } \underline{2.39}$	B1	1.1b
		(2)	
(e)	(i) $[D \sim \text{Po}(2.4) \quad P(D \leq 4)]$ $= 0.9041... \quad \text{awrt } \underline{0.904}$	B1	1.1b
	(ii) Since n is large and p is small/mean is approximately equal to variance	B1	2.4
		(2)	
(10 marks)			
Notes:			
(a) M1: For $P(X=2) \times P(Y=2)$ from $X \sim \text{Po}(2.6)$ and $Y \sim \text{Po}(1.2)$ i.e. correct models (may be implied by correct answer) A1: awrt 0.0544			
(b) M1: For combining Poisson distributions and use of Po('3.8') (may be implied by correct answer) A1: awrt 0.205			
(c) M1: For setting up a new model and attempting mean of Poisson distribution (may be implied by correct answer) A1: awrt 0.648			
(d)(i) B1: For 2.4			
(d)(ii) B1: For awrt 2.39			
(e)(i) B1: For awrt 0.904			
(e)(ii) B1: For a correct explanation to support use of Poisson approximation in this case			

Question	Scheme	Marks	AOs
4(a)	(i) $P(X = 1) = 0.34523\dots$ awrt 0.345	B1	1.1b
	(ii) $P(X \leq 4) = 0.98575\dots$ awrt 0.986	B1	1.1b
		(2)	
(b)	$\frac{(0 \times 10) + 1 \times 16 + 2 \times 7 + 3 \times 4 + 4 \times 2 + (5 \times 0) + 6 \times 1}{40} = 1.4^*$	B1*cs0	1.1b
		(1)	
(c)	$r = 40 \times '0.34523\dots'$ $s = 40 \times '1 - 0.986\dots'$	M1	3.4
	$r = \underline{\mathbf{13.81}}$ $s = \underline{\mathbf{0.57}}$	A1ft	1.1b
		(2)	
(d)	H ₀ : The Poisson distribution is a suitable model H ₁ : The Poisson distribution is not a suitable model	B1	3.4
	[Cells are combined when expected frequencies < 5] So combine the last 3 cells	M1	2.1
	$\chi^2 = \sum \frac{(O - E)^2}{E} = \frac{(10 - 9.86)^2}{9.86} + \dots + \frac{(7 - (4.51 + 1.58 + 0.57))^2}{(4.51 + 1.58 + 0.57)}$	M1	1.1b
	awrt 1.1	A1	1.1b
	Degrees of freedom = 4 – 1 – 1 = 2	B1	3.1b
	(Do not reject H ₀ since $1.10 < \chi^2_{2,(0.05)} = 5.991$). The number of mortgages approved each week follows a Poisson distribution	A1	3.5a
		(6)	
(11 marks)			
Notes:			
(a)(i) B1: awrt 0.345			
(a)(ii) B1: awrt 0.986			
(b) B1*: For a fully correct calculation leading to given answer with no errors seen			
(c) M1: For attempt at r or s (may be implied by correct answers) A1ft: For both values correct (follow through their answers to part (a))			
(d) B1: For both hypotheses correct (lambda should not be defined so correct use of the model) M1: For understanding the need to combine cells before calculating the test statistic (may be implied) M1: For attempt to find the test statistic using $\chi^2 = \sum \frac{(O - E)^2}{E}$ A1: awrt 1.1 B1: For realising that there are 2 degrees of freedom leading to a critical value of $\chi^2_{2,(0.05)} = 5.991$ A1: Concluding that a Poisson model is suitable for the number of mortgages approved each week			